Review: Function Definition - 9/12/16

1 Definition of a Function

Definition 1.0.1 *A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.*

The "exactly one" part of the definition means that the function can only send each x to one $f(x)$. This manifests itself graphically by the vertical line test.

Definition 1.0.2 *The Vertical Line Test: A curve is the graph of a function if and only if no vertical line intersects the curve more than once.*

2 Domain and Range

Definition 2.0.3 *The **domain** of a function f is the set of values for which the function is defined.*

Example 2.0.4 *What is the domain of $f(x) = x^2$? It is all real numbers, denoted $(-\infty, \infty)$.*

Example 2.0.5 *What is the domain of $g(x) = \frac{1}{x}$? It is $(-\infty, 0) \cup (0, \infty)$.*

Example 2.0.6 *What is the domain of $f(x) = \sqrt{x}$? It is $[0, \infty)$.*

When looking for the domain of a function you should **find the values for which the function is not defined** (aka where the function doesn't make sense). Things to check:

1. Bottom of fraction
2. Inside of square root
3. Arbitrary domain specification

Definition 2.0.7 *The **range** of a function f is the set of all possible values of $f(x)$.*

Example 2.0.8 *What is the range of $f(x) = x^2$? It is $[0, \infty)$. What is the range of $g(x) = x^2 + 2$? It is $[2, \infty)$.*

Example 2.0.9 *What is the range of $f(x) = \frac{1}{x}$? It is $(-\infty, 0), (0, \infty)$.*

Example 2.0.10 *What is the range of $f(x) = \sqrt{x}$? It is $[0, \infty)$.*

Practice Problems

Find the domain of the following functions:

1. $f(x) = \frac{1}{x^2+13x+32}$.

2. $g(x) = \sqrt{x^2 - 4}$.

3. $h(x) = x^3 - 2$.

Find the range of the following functions:

1. $f(x) = (x+2)^2$.

2. $g(x) = \frac{1}{x-7}$.

3. $h(x) = \sqrt{x} - 2$.

3 Operations on Functions

We can add, subtract, multiply, and divide functions.

Example 3.0.11 *For the following examples, let* $f(x) = x^2$, $g(x) = \sqrt{2x+3}$.
$(f+g)(x) = x^2 + \sqrt{2x+3}$.
$(f-g)(x) = x^2 - \sqrt{2x+3}$
$(fg)(x) = x^2(\sqrt{2x+3})$
$(\frac{f}{g})(x) = \frac{x^2}{\sqrt{2x+3}}$

There's an extra operation that we can do with functions that we can't do with numbers. This operation is called composition.

Definition 3.0.12 *Given two functions* f *and* g*, the* ***composition*** *of* f *and* g*,* $f \circ g$*, is defined by* $(f \circ g)(x) = f(g(x))$.

Example 3.0.13 *Let* $a(x) = \frac{1}{x}$ *and* $b(x) = x^3 + 7$*. Then* $(a \circ b)(x) = \frac{1}{x^3+7}$ *and* $(b \circ a)(x) = \left(\frac{1}{x}\right)^3 + 7$.

Practice Problems
Let $f(x) = \sqrt{x}$, $g(x) = x^2$. Write down the equations and find the domain for the following functions:

1. $f(x)$

2. $g(x)$

3. $(f+g)(x)$

4. $(f-g)(x)$

5. $(fg)(x)$

6. $(\frac{f}{g})(x)$

7. $(f \circ g)(x)$

8. $(g \circ f)(x)$

4 Sequences

Definition 4.0.14 *A **sequence** is an ordered list of numbers.*

Example 4.0.15 *There are three ways to write the sequence* $\{1, 2, 3, 4, 5, \dots\}$*:*

1. $\{1, 2, 3, 4, 5, \dots\}$
2. $\{n\}_{n=1}^{\infty}$
3. $a_n = n$

If we're talking about an arbitrary sequence, then we write $\{a_n\}_{n=1}^{\infty}$.

Example 4.0.16 *Write out the first few terms of each sequence:*
$\{a_n\} = \{\frac{(-1)^n}{n^2}\}$
$a_1 = \frac{-1}{1}$, $a_2 = \frac{1}{4}$, $a_3 = \frac{-1}{9}$

$b_n = 2b_{n-1} + b_{n-2}$, *with* $b_1 = 2$ *and* $b_2 = 4$
$b_3 = 10$, $b_4 = 24$, $b_5 = 58$

Practice Problems
Write out the first few terms of the following sequences:

1. $\{\frac{n}{(n+1)^2}\}_{n=1}^{\infty}$
2. $\{\frac{1}{n^3}\}_{n=1}^{\infty}$
3. $a_n = 3a_{n-1}$ with $a_1 = 1$.

Find the formula for the following sequences:

1. $\{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \dots\}$
2. $\{\frac{1}{6}, \frac{2}{8}, \frac{3}{10}, \frac{4}{12}, \dots\}$